Metropolitan Washington Council of Governments
National Capital Region Transportation Planning Board

Current Use of Traffic Simulation and Dynamic Traffic Assignment (DTA) Models by MPOs

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VHB

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Executive Summary

The Metropolitan Washington Council of Governments, National Capital Region Transportation Planning Board (TPB) engaged Vanasse Hangen Brustlin (VHB) to review the current use of traffic simulation and Dynamic Traffic Assignment (DTA) models among Metropolitan Planning Organizations (MPOs) in the United States.

Traffic simulation models can generally be considered to fall into three categories: microscopic, microscopic with DTA capability, and mesoscopic DTA. A mesoscopic traffic model represents traffic as platoons of vehicles traveling through the network. Mesoscopic models are generally associated with DTA, however, there are a number of mesoscopic models without DTA capability that are used for sketch planning purposes where schedule and budget do not allow for detailed microsimulation. By contrast, a microscopic traffic model is the most detailed representation of a vehicle's movement through a network, often on a second-by-second (or smaller) basis. Because of its high level of detail, traffic simulation models are able to produce an animation of individual vehicles moving through the network. This memo summarizes the two categories of traffic simulation models as well as a number of mesoscopic simulation-based DTA models that have recently been introduced by a number of vendors.

While a number of MPOs have reported the use of a variety of traffic simulation software packages, only a few are currently exploring the use of mesoscopic DTA models, with no MPOs reporting having utilized these tools to date.

The use of traffic simulation models and Dynamic Traffic Assignment models are increasing in popularity among MPOs. The results of this research effort reflect this trend, as there are a significant number of traffic simulation models on the market and a growing number of DTA models as well. Table 1 summarizes the most popular software and current capabilities; it is important to note that all of the models listed are under constant development which means the current capabilities could and most likely will change in the future.

Table 1: Simulation and DTA Model Capabilities

Application							
Traffic Simulation Models	AIMSUN	CORSIM	Cube Dynasim	Paramics	SimTraffic	TransModeler	VISSIM
Pretimed Signals	Y	Y	Y	Y	Y	Y	Y
Actuated Signals	Y	Y	Y	Y	Y	Y	Y
Unsignalized Intersections	Y	Y	Y	Y	Y	Y	Y
Cars, Trucks	Y	Y	Y	Y	Y	Y	Y
Bus Routes, Bus Stops	Y	Y	Y	Y	N	Y	Y
On Street Parking	Y	Y	Y	Y	N	Y	Y
Pedestrians	Y	L	Y	Y	Y	Y	Y
Car Pools	Y	Y	Y	Y	N	Y	Y
Bus and Carpool Lanes	Y	Y	Y	Y	N	Y	Y
Freeway	Y	Y	Y	Y	L	Y	Y
Ramp Metering	Y	Y	Y	Y	N	Y	Y
Roundabouts	Y	L	Y	Y	Y	Y	Y
Right Turn Islands	Y	L	Y	Y	Y	Y	Y
Temporary Events	Y	Y	Y	Y	N	Y	Y
Transit Priority	Y	N	Y	Y	N	Y	Y
Light Rail	Y	N	Y	Y	N	Y	Y
Toll Plazas	Y	L	Y	Y	L	Y	Y
Variable Message Signs	Y	N	Y	Y	N	Y	Y
Dynamic Assignment	Y	N	N	Y	N	Y	Y
DTA Models	Dynamec	Dynasmart-P	Cube Avenue				
Queuing Evaluation	Y	Y	Y				
HOT Lanes	Y	Y	Y				
Variable Message Signs	L	Y	N				
Evacuation Plans	Y	Y	Y				
Special Event Planning	Y	Y	Y				
Ramp Metering	N	Y	Y				
Traffic Signals	Y	Y	Y				

Y=Yes; L=Limited; N=No

The research illustrates that TPB has a number of traffic simulation and DTA models to select from. There are a number of factors to consider when selecting a software package, including existing staff capability (training implications), potential staff recruitment (the more popular the software, the easier it will be to locate experienced users), potential applications (i.e., does TPB want to evaluate LRT, Intelligent Transportation Systems [ITS], and/or signal timing strategies), and institutional history (i.e., if TPB uses CUBE, one would expect that it would require less effort and training to utilize Dynasim and Avenue than TransModeler or Dynasmart).

Traffic Simulation versus DTA (Mesoscopic)

Traffic simulation models have been used for project planning and traffic operational studies for several decades. Recent advances have added transit priority and a variety of ITS applications to the list of topics that can now be evaluated in detail using these tools.

The challenge with the use of traffic simulation models lies in the amount of labor involved in data collection, network coding, and network calibration, as simulation models require detailed turning movement volume data, traffic signal timing settings, and detailed lane geometries, down to the lengths of turn bays.

While the output of traffic simulation models, particularly ones with three-dimensional (3D) animation and visualization capabilities, are popular among high-level MPO decision-makers and board members, the amount of labor required of technical staff and the associated budget involved makes it difficult to develop these services and maintain them in the long run. These constraints have led to the development of a new breed of models, mesoscopic DTA models that operate at a scale between the typical regional travel demand forecasting models (macroscopic) and traffic simulation models (microscopic).

The mesoscopic DTA models utilize the theory of user equilibrium assignment combined with time dependent origin-destination (OD) matrices to give audiences a richer representation of traffic conditions at the regional and sub-regional level than a regional static planning model while simultaneously reducing the amount of effort required for network coding and calibration when compared to microscopic traffic simulation models. A typical DTA model can utilize the network geometry directly from the regional model, and the user can add additional details such as the number of turn lanes and signal timing data in the areas of greatest interest while using default values for the remaining portions of the network. Since the DTA models are simulation-based, queuing associated with capacity constraints in the network can be readily observed, unlike in static planning models.

Traffic Simulation Models

SimTraffic

SimTraffic is a microscopic traffic simulation model produced by Trafficware. SimTraffic was designed to work with Synchro as its interface, and Synchro serves the dual purpose of performing traffic signal optimization as well, making it perhaps the most widely used simulation package in the U.S. SimTraffic can be used to evaluate arterials, particularly pretimed and actuated traffic signals, unsignalized intersections, pedestrians, roundabouts, and right turn lanes. SimTraffic has the capability of simulating freeways, but the software is limited in that there is no explicit freeway link inherent in the software meaning that the car following and lane changing methodologies that are borrowed from CORSIM are based on the arterial methodologies from the NETSIM component of CORSIM which are not identical to the methodologies used for freeways. This limitation is often overcome to some degree by adjusting the parameters of the Synchro links to reflect speed and merging conditions that would prevail on freeways, but the model often struggles in highly congested conditions partially because of a lack of true freeway car following and lane changing methodologies.

SimTraffic is a relatively easy-to-use tool adding to its popularity among practitioners. The network coding occurs in Synchro and requires less labor than CORSIM and other simulation packages as Synchro nodes are created automatically by overlapping links, eliminating the need to code both links and nodes as in CORSIM. The traffic signal timing coding requires a more experienced traffic engineer as does the calibration process, but Synchro explicitly optimizes both isolated traffic signals as well as signal systems whereas CORSIM requires an experienced traffic engineer who can develop the optimized timing plans manually or through numerous iterations with the model, or by interfacing between CORSIM and SimTraffic. As with the majority of simulation software, Synchro/SimTraffic networks can be coded over aerial photography which allows for quicker base network coding.

Synchro/SimTraffic is also popular among MPOs. It is used by over 20 MPOs including 3 MPOs similar to TPB.

CORSIM

CORSIM is one of the oldest and most popular traffic simulation models in current use. The software was developed by the Federal Highway Administration (FHWA) to support operational analysis and has steadily advanced over the past several decades. Like SimTraffic, CORSIM can model arterials, though signal timing optimization is not a part of the software; signal timing adjustments have to be made manually which adds to the level of effort for arterial analysis when compared to SimTraffic. CORSIM includes car following and lane changing methodologies for both freeway and arterial segments, making it more appropriate for freeway analysis than SimTraffic. CORSIM can be used to evaluate a variety of traffic control strategies, including stop/yield signs, actuated and pretimed traffic signals, and ramp metering. Moreover, CORSIM can simulate bus routes and bus stops including dwell times, on-street parking, pedestrians (using delay factors; not modeled explicitly), carpools, HOV lanes, bus lanes, limited roundabout analysis (all links in a roundabout must be longer than 50 feet which can be difficult to achieve), HOV lane bypass at ramp meters, incidents, weigh stations, toll booths, and airport loading/unloading zones. It should be noted that a number of the applications, such as toll booths and airport loading/unloading require experienced traffic engineers to ensure the model is replicating field conditions in a reasonable manner.

CORSIM historically has been a labor-intensive software package, though much progress has occurred in this regard recently, particularly with the introduction of TRAFED, the graphical editor that allows links and nodes to be coded over network maps or aerial photography. TRAFED also improved on many of the glitches discovered in ITRAF, the previous graphical interface; this dramatically reduces the number of fatal errors in a coded network which require an experienced CORSIM user to debug.

The software requires inputs such as traffic control and signal timing data, roadway and intersection geometries, and demand data; either through explicit turning movement counts, turning movement percentages, or limited capability for OD table input. Depending on the environment simulated, additional inputs may be required including truck percentages, bus routing information, bus dwell times at stops, pedestrian counts, and number of parking maneuvers. Typically, an entry-level engineer or planner can be utilized for basic network coding, such as roadway geometry and intersection control; whereas a number of advanced parameters require a more experienced traffic engineer who has an understanding of traffic signal timing and traffic flow theory. This is particularly important for calibration purposes, which requires the adjustment of factors such as driver aggression and familiarity.

Based on a recent MPO survey, CORSIM is used by over 40 MPOs, including 12 very large MPOs similar to TPB.

Cube Dynasim

Cube Dynasim is a relatively new simulation model developed by Citilabs, the developers of TP+, Cube, and TRANPLAN. Dynasim was developed to provide a relatively seemless transition between Cube regional planning models and detailed traffic simulation. Dynasim can be used to evaluate a variety of scenarios, including arterials, expressways, HOV lanes, toll plazas, ramp metering, taxi stands, transit priority, ITS, evacuation plans, truck terminals and advanced signal systems and technologies.

Cube Dynasim's required inputs include a highway and if applicable, transit network, which can be coded manually using aerial photography or maps, or networks can be extracted directly from Cube or Viper travel demand model networks. Other inputs include signal timing data which can be input manually, or imported from Synchro, demand data in OD format, general traffic control, and bus routing information. The integration with Cube Voyager and TP+ reduces the amount of effort required for coding networks, though a lot of additional coding would still be required including signal timing and detailed intersection geometries that are necessary to calibrate the Dynasim model.

COG/TPB staff has purchased a copy of both Cube Dynasim and Cube Avenue (DTA). Although staff has not had the time to work with the DTA module, staff has done some work simulating traffic around I-395 and South Eads Street in Arlington County using Dynasim. Based on this initial work, staff had the following observations: First, developing a simulation in Dynasim is very labor intensive and time consuming. Although it is true that you can export a sub area network from TP+/Voyager to Dynasim, the exported network then needs a substantial amount of additional coding detail and clean up work. Consequently, it may actually be quicker to simply manually re-code all of the study area links and intersection controls directly in Dynasim, which, in itself is no small task. Second, staff felt that the Dynasim software had a number of bugs in it, which Citilabs worked diligently to correct, but nonetheless, ended up making it difficult to work in a timely manner. Third, acquiring quality traffic counts needed to calibrate the simulation is often difficult or impossible. Fourth, another time consuming feature of developing these simulations is acquiring the AutoCAD files of road geometries from state and local governments. It is important to note, that a number of the limitations pointed out by COG/TPB staff, such as the availability of traffic counts and coding effort would apply to all of the simulation models researched.

Cube Dynasim is currently used by one MPO based on a recent survey; however, this number is expected to grow in the future due to Dynasim's integration within the Cube suite of software which is used for regional planning by a large number of MPOs in the U.S, including TPB. There is currently little experience with the software, so the jury is still out on how well it calibrates, etc.

Traffic Simulation Models with DTA Capability

Paramics

The United Kingdom-based company Quadstone is the manufacturer of Paramics, a suite of microscopic simulation modules in an integrated platform. Paramics is fully scaleable and designed to handle scenarios as wide-ranging as a single intersection to a congested freeway or the modeling of an entire city's traffic system. Paramics can be used to evaluate transit priority, arterials, congested freeways, HOV, ITS, including ramp metering, variable message signs, route control, lane usage, and freeway speed control, parking, incidents, and work zones.

Paramics can translate a variety of common files including Synchro, CORSIM, and Cube/TP+. This helps reduce the labor and learning curve associated with network coding and debugging; however, it is important to note that Paramics uses only OD information to simulate demand. This would potentially add to the calibration effort as typically link speed is one of the primary calibration adjustments. In the case of a Paramics assignment whether static or dynamic, link speed is also a variable in route choice, meaning that link speed adjustments would need to occur in an iterative fashion until the demand pattern matches observed traffic volumes in the field. This could also lead to some instances where the link speed after calibration does not closely match field conditions.

There are currently less than a handful of MPOs using Paramics based on a recent survey. These MPOs are concentrated on the East Coast where Quadstone has its U.S. headquarters.

VISSIM

Over the past decade, VISSIM has become one of the more popular traffic simulation software packages in the U.S., particularly in the context of light rail and bus rapid transit (BRT) evaluation due to its ability to model transit priority. Developed by the German company PTV AG and marketed here through its U.S. subsidiary, PTV America, VISSIM is a microscopic, behavior-based multi-purpose traffic simulation program. VISSIM can be used to evaluate arterials, congested freeways, transit priority, traffic management systems such as alternative route control, traffic flow control, toll roads, access control, HOV and HOT lanes, feasibility analysis of large networks with alternative route choice using dynamic assignment, capacity analysis of toll plazas and border control facilities, traffic calming, parking, parallel vehicle flows (e.g. cars and motorcycles) driving in the same lane as well as overtaking vehicles inside wide lanes, and NEMA and Type 170 signal controller interfaces for real-time evaluation.

VISSIM uses a link-connector system to lay out networks. This allows for greater flexibility with regards to evaluating complicated intersections and roadway/transit networks, but adds significantly to the effort required for network coding. In response to this, VISSIM recently introduced a Synchro interface which allows Synchro files to be translated directly into VISSIM with minimal modifications. VISSIM can also read data in GIS format which can reduce the effort required for network coding; however, since the software gives the user greater control over the network coding and hence, calibration procedures than other software such as

Synchro/SimTraffic and CORSIM, VISSIM applications typically require a more experienced traffic engineer to ensure model accuracy and consistency.

Reflecting its increasing popularity in the U.S., VISSIM is used by approximately two dozen MPOs currently, including a dozen similar in size to TPB.

AIMSUN

AIMSUN is a microscopic traffic simulation model that has been compared favorably by practioners to VISSIM in the past. AIMSUN can simulate urban networks, freeways, arterials and any combination thereof. It has been designed and implemented as a tool for traffic analysts to help traffic engineers in the design and assessment of traffic systems. It has proven to be very useful for testing new traffic control systems and management policies including, adaptive traffic control systems such as SCATS, transit priority, Advanced Traffic Management Systems (ATMS) including ramp metering and variable message signs, vehicle guidance systems, and incidents.

AIMSUN also has DTA capabilities. The simulator is able to model the drivers' reasoning for route selection before and during the trip. It includes four different algorithms to model dynamic route choice, a function editor to allow the specification of cost functions, and the option of considering the costs from historical routes and/or considering the driver's memory. A variety of drivers will use different criteria: from always sticking to the same path to changing their path according to advice from a guidance system or traffic conditions.

According to a recent survey, no MPOs reported using AIMSUN.

TransModeler

TransModeler is a traffic simulation model developed by the Boston-area based Caliper Corporation, the manufacturers of TransCAD. TransModeler has the capability to model mixed freeway and arterial networks, HOV lanes, bus lanes, toll facilities, evacuation plans, work zones, traffic signal systems, traffic signal preemption, lane use signs and flexible variable message signs, ramp metering effects on freeway and adjacent urban streets, the impact of real-time traffic information on dynamic driver rerouting, and transit priority.

TransModeler has a unique GIS architecture that integrates traffic simulation models with a GIS that has been extended to store, maintain, and analyze transportation and traffic data. This allows for the storage of information such as traffic counts, lanes, and speeds which becomes a useful database for future studies.

TransModeler is also unique in that it can simulate at the microscopic, mesoscopic, and macroscopic levels, including hybrid simulations in which microsimulation can be intermixed with mesoscopic and macroscopic simulation on any network segments. This allows the network of greatest interest to be simulated at the micro level and others at the mesoscopic and/or macroscopic scale which makes it possible to simulate very large networks with modest computing power.

TransModeler is integrated with TransCAD allowing for integrated travel demand and traffic modeling. Travel demand forecasts can be subjected to more detailed operational analysis with the use of embedded matrix estimation procedures (adjusting the OD table to match existing count data). Conversely, traffic simulation results can be fed back to the travel demand model for improved destination and mode choice.

TransCAD can be used to run a Stochastic User Equilibrium or Dynamic User Equilibrium assignment to generate congested link travel times, flows, and turning movements as input to the TransModeler route choice models.

To help aid the learning curve associated with TransModeler, CORSIM and SimTraffic files can be imported, though it is unclear how much additional effort may be required to further format these files prior to running TransModeler. The required inputs are similar to other traffic simulation software and include detailed lane geometries, traffic control data, demand data including vehicle and truck counts or OD matrices, pedestrian counts, and transit routing information. In this regard, experienced traffic simulation modelers should have a relatively modest learning curve with regards to basic data inputs. The GIS based scenario development implies that the user would also need to develop a basic understanding of GIS, which is unique to TransModeler.

TransModeler is a relatively new entry into the traffic simulation arena. Hence, no MPOs responded as having used the software to date; however, based on conversations with Caliper, several MPOs who use TransCAD for regional travel demand forecasting are exploring the use of TransModeler.

Dynamic Traffic Assignment Models

Dynameq

Dynameq, developed by the Canadian company INRO (the developers of EMME/3), is an equilibrium DTA model for use on large congested networks. Dynameq enables planners to evaluate congested network scenarios with dynamic equilibrium benchmarks, a time varying version of the same well-understood equilibrium assignments used in static analysis for years. Dynameq's equilibrium traffic assignment results represent user optimal network conditions that are immediately useful as an upper-bound on network performance.

Traffic phenomena that trigger congestion are modeled explicitly, including signals, conflicting movements at intersections, lane permissions for turning movements and vehicle classes, and weaving. Each vehicle travels along a particular lane, performs lane changes where appropriate, and crosses signalized and unsignalized intersections. Large networks tend to be more data-intensive. Dynameq is designed with a minimal set of meaningful model parameters to get the model up and running as quickly as possible. The user can focus data collection and network coding effort to the parts of the network that need it most, and use link and intersection default settings, for less critical parts of the network. One can use constant demand extracted from static planning models, or separate the demand matrix into time slices.

The user can draw insight from simulation results using a variety of analysis tools, and communicate results to decision-makers. Decision makers can see the big picture with animated network-scale results to identify congestion patterns and assess the extent of congestion with animated plots of lane-by-lane queues.

The current maximum network size consists of 10,000 links, 5000 intersections, and 1000 transportation analysis zones. Dynameq is used to evaluate lane closures, infrastructure expansion at the sub-regional level, Managed Lanes, HOT Lanes, pre-timed signal control, and incidents.

As Dynameq is manufactured by INRO, the developers of EMME/3, EMME/3 users would require minimal training to use Dynameq. Dynameq, like other DTA models, does not require the level of detailed inputs that traffic simulation models require, which also reduces the amount of labor involved in network coding. Dynameq has not developed an interface as of yet to read in Synchro, CORSIM, and/or Cube/TP+ networks which would make it more cumbersome for experienced users of those software packages to implement Dynameq.

Dynameq is currently not used by any of the U.S. MPOs that responded to a recent survey. This is consistent with the limited use of EMME/3 by those MPOs.

Dynasmart-P

Dynasmart-P uses mesoscopic simulation combined with DTA to model the evolution of traffic flows in a traffic network, which result from the travel decisions of individuals. The model is also capable of representing travel decisions of travelers seeking to fulfill a chain of activities at different locations in a network over a given planning horizon.

Dynasmart-P was developed by the University of Maryland, College Park, in concert with FHWA to address the growing need to evaluate Intelligent Transportation Systems (ITS) in the regional planning context. Dynasmart can provide dynamic traffic assignment methods for traditional planning analyses, assess the impacts of ITS technologies, such as dynamic message signs, ramp meters, and in-vehicle guidance systems; assess the impacts of different traffic operations and control strategies, evaluate regional work zone management, evaluate incident management and special event management strategies, and evaluate congestion-pricing schemes.

However, it is important to note that Dynasmart-P cannot model detailed traffic maneuvers, such as car-following, lane-changing, and weaving operations which would still require microscopic analysis. In addition, there is currently limited transit and inter-modal modeling capabilities, though future versions will be better able to perform this type of modeling.

While Dynasmart-P is a new software package with limited applications to date, it has been used in the following efforts:

- Develop traffic management strategies for major highway reconstruction projects in Zwolle, a city in the Netherlands.

- Evaluate downtown El Paso, TX traffic and environmental impacts of one and two-way traffic flow reconfigurations. This project used a combination of Dynasmart-P and CORSIM.

- Undertake a pilot study to apply DTA as a part of the regional four-step modeling process in the El Paso, TX region.

Dynasmart can translate networks from Cube/TP+, CORSIM, and most GIS formats; this allows large networks to be readily converted into Dynasmart format. Dynasmart can either use default traffic control settings or actual signal timing data which allows for greater flexibility in the amount of labor required as the user can focus their coding efforts on the areas of greatest interest while using default values for the rest. However, it is important to note that using the default traffic control can lead to similar issues as what occurred with TRANSIMS where it was discovered that traffic control and signal timing had a much greater impact on route choice than the researchers expected.

Another area of potential concern with regards to Dynasmart is the lack of transparency in the OD estimation process. To date, all of the efforts involving Dynasmart have required that the OD estimation occur at the University of Maryland. For Dynasmart to become more of a mainstream software package, the OD estimation procedures utilized by the University of Maryland will need to be shared with other users, or other matrix estimation software such as those developed by Citilabs and others will need to be tested on Dynasmart networks.

There are currently a handful of MPOs testing Dynasmart, primarily in the areas of evacuation planning and regional ITS planning.

Cube Avenue

Cube Avenue is a mesoscopic model developed by Citilabs, the developers of Cube, TP+, and Voyager. By explicitly modeling time, Cube Avenue can be used for studies comparing policies for alleviating peak period congestion, such as variably priced toll lanes, as well as evaluating the effectiveness of emergency evacuation plans. Cube Avenue can also be used to quantify impacts of upstream traffic congestion, measure queuing at intersections and merge points in a network, isolate secondary impacts from one intersection to another, ITS strategies such as HOT lanes and ramp metering, emergency evacuation plans and strategies, special event planning, and traffic control, including traffic signals, roundabouts, and stop-controlled intersections.

Cube Avenue works with conventional Cube/TP+ job scripts and networks which minimizes the learning curve for current users of Cube. The networks and associated OD tables can be extracted directly from regional networks, further reducing the amount of labor associated with network coding. Furthermore, as this is a mesoscopic model, the level of detail associated with network representation is less than that of traffic simulation models.

Cube Avenue was recently released by Citilabs. Hence, no MPOs responded as having used this software in a recent survey, though it is likely that a significant number of MPOs will be evaluating and/or utilizing this software in the future based on the large number of MPOs in the U.S. who use the Cube suite of software for regional travel forecasting.

TRANSIMS

TRANSIMS is an agent-based simulation system capable of simulating the second-by-second movements of every person and every vehicle through the transportation network of a large metropolitan area. It was developed by the Los Alamos National Laboratory.

It consists of mutually supporting simulations, models and databases. By employing advanced computational and analytical techniques, it creates an integrated environment for regional transportation system analysis.

TRANSIMS is designed to give transportation planners more accurate, complete information on traffic impacts, energy consumption, traffic congestion, land use planning, traffic safety, intelligent vehicle efficiencies, and emergency evacuation.

TRANSIMS has the capability to analyze traffic over the entire transportation network of a metropolitan area, including local streets and highway ramps, compute precise speed and acceleration information for every single vehicle at any second of the day, and provide second-by-second information allowing for a much more precise analysis of time-of-day effects.

While TRANSIMS is a very powerful modeling platform in theory, when applied to the Portland region it was discovered that the model requires detailed roadway geometry, signal timing, and phasing data to accurately model route choice; acquiring this data for an entire metropolitan area the size of the TPB region would be a labor intensive effort with significant costs associated with it. Moreover, modeling a region the size of Metropolitan Washington to this level of detail would require significant computer processing capabilities and even with this, it would likely take days for the model to run, making it impractical for most TPB applications.

Conclusions

The simulation and/or DTA software selected and implemented by TPB should reflect existing staff and consultant capabilities as well as provide new and/or better solutions to the most pressing modeling questions that TPB faces. For example, peak spreading and managed lanes are two areas where the existing regional model has limitations, and DTA would theoretically provide better answers because of its ability to explicitly model time as well as capture impacts of traffic control and queuing, which all relate to peak spreading. Likewise, Express Toll Lanes (ETL) require the ability to model congestion over time, which would require a DTA model to do this type of analysis at the regional level or a traffic simulation model with DTA capability to evaluate these types of strategies at the corridor level. In addition to these topics, traffic simulation models could be used to evaluate corridors as a part of the federally-mandated congestion management system (CMS) program.

COG/TPB has traditionally used macroscopic traffic assignment methods, such as static user equilibrium traffic assignment, to carry out its regional transportation planning activities. As this memo points out, the used of traffic microsimulation models require a lot of data and coding effort, which would preclude them as a practical tool for regional planning purposes, unless specific corridors are being evaluated in detail. The more appropriate tool for regional planning purposes, particularly in the context of HOT/Managed Lanes analysis and regional ITS planning would be a mesoscopic DTA model which would explicitly model peak period demand over time and illustrate the queuing affects associated with HOT lanes and other roadway elements. The mesoscopic DTA models do not require the level of detail that microsimulation models require; which reduces the coding effort and allows for a more seamless transition between the regional model networks and the mesoscopic DTA model networks. Given that COG/TPB staff uses Cube for regional planning efforts, Cube Avenue would be the most practical mesoscopic DTA model to use for HOT/Managed lanes analysis. Dynasmart has limited technical support which makes it very difficult for staff to learn and implement the software effectively, and Dynameq has limitations with regards to network size, making it difficult if not impossible to utilize in large metropolitan planning regions such as Washington, D.C.

Moving forward, TPB may want to pursue a pilot study where DTA is used as the fourth step of the modeling process and conduct a screenline and corridor level validation to determine if the DTA assignment is indeed an improvement over the existing static equilibrium process. If the results are positive based on this test study, then it is recommended that TPB test DTA in the context of ETL and HOT lanes as well as peak spreading.

Appendix -- Detailed software methodologies

CORSIM Methodologies

The freeway component (FRESIM) in CORSIM uses the Pitts car-following model which is based on the distance headway between vehicles. The objective function of the model is dependent on the lead vehicle length, driver sensitivity of the following vehicle, speed of the following vehicle at time *t*, speed of the lead vehicle at time *t*, and a calibration constant defined by the user. The calibration constant affects vehicle acceleration and deceleration rates which in turn affects the headways that can be maintained between vehicles.

The arterial component (NETSIM) of CORSIM uses car-following logic where the independent or lead vehicle attempts to maintain free-flow speed and the follower avoids collisions with the leader.

The lane changing logic is broken out into three categories:

- Mandatory, which is based on acceptable risks for the driver making the lane change. This logic is used for lane drops, merging, exits, and lane blockages.

- Discretionary, which is based on driver behavior (aggressive vs. non-aggressive).

- Anticipatory, which is applied before on-ramps to allow vehicles to enter the freeway.

Lane changes in general are dependent on gap acceptance models where an acceptable minimum gap between vehicles in the target lane is required to accommodate a lane changer.

Similarly, the gap acceptance models used in NETSIM to simulate stop/yield conditions and permissive turns are based on the time to travel from conflicting point to the opposite stop line (or stop bar). This model is also a function of driver composition and is one of the calibration components in CORSIM, i.e. aggressive driver populations will accept smaller gaps than non-aggressive populations.

SimTraffic Methodologies

In general, SimTraffic uses the same driver and vehicle characteristics as the NETSIM component of CORSIM. With regards to car-following, SimTraffic uses a formula that has vehicles track leaders at a fixed headway. The headway is dependent on speed, driver type, and link characteristics. The acceleration rates used in SimTraffic are identical to NETSIM and the deceleration rates used in SimTraffic are very close to those used in NETSIM.

SimTraffic also has similar lane change logic to NETSIM. The vehicles will complete a lane change when the next lane is clear. To be clear, both the changing vehicle and the vehicle behind must not obtain a deceleration rate above the threshold using the car-following formulas.

In SimTraffic a vehicle can be stopped in the middle of a lane change and block 2 lanes. SimTraffic's lane changes tend to be more disruptive than NETSIM because the vehicles require a forward movement to complete the lane change where in NETSIM they do not.

In SimTraffic, gap acceptance is based on the type of turn made and the length of the turning path. The gap times in SimTraffic are more consistent with the amount of time required to complete the turn and are towards the high end of the gap times in NETSIM. In general, SimTraffic will accept fewer gaps but give safer operation.

Cube Dynasim Methodologies

Dynasim uses methodologies developed by Kazi Ahmed at the Massachusetts Institute of Technology (MIT). The car-following model is based on two regimes, the free-flow regime where a driver is assumed to try to attain his/her speed and the car-following regime where the driver is assumed to follow his/her leader. A probabilistic model that is based on a time headway threshold is used to determine the regime the driver belongs to. Heterogeneity across drivers is captured through the headway threshold and reaction time distributions. The parameters of the car-following and free-flow acceleration models along with the headway threshold and reaction time distributions are jointly estimated using the maximum likelihood estimation method.

The lane changing decision process is modeled as a sequence of three steps: decision to consider a lane change, choice of a target lane, and gap acceptance. Since acceptable gaps are hard to find in heavily congested traffic, a forced merging model that captures forced lane changing behavior and courtesy yielding is developed. A discrete choice model framework is used to model the impact of the surrounding traffic environment and lane configuration on drivers' lane changing decision process.

Paramics Methodologies

The lane changing methodology used in Paramics is prioritized into two levels, urgent and non-urgent. Within Paramics, a driver will attempt to execute a lane change maneuver as a response to either a single urgent stimulus or a series of five contiguous and consistent non-urgent stimuli produced by unsuitable transient conditions.

An urgent stimulus is generated if a driver finds itself outside its target range of lanes. Near a hazard, the target range is controlled by the number of lanes available on the exit link appropriate to the driver's choice of route. At all times, the target range is adjusted subject to the behavior patterns associated with the driver and vehicle type; a higher level of aggression causes a driver to move to the outer (higher speed) lanes, a higher level of awareness causes a vehicle to adopt the target lane for an impending turn sooner.

An urgent stimulus is also generated if the vehicle caught in a stationary line of traffic (as a result of an incident for example).

A non-urgent stimulus can be generated for a number of conditions, which are themselves prioritized as follows:

- Move in or out because of constraints imposed by a fixed physical object such as a ramp joining, or a climbing lane.

- Move in or out as suggested by free-flow lane-changing model. This can be defined by the user, or the standard free-flow model can be used.

- Move in or out on an urban road in such a way as to spread the total demand over the available road space. In the absence of other stimuli, this prevents false congestion from building up.

Note that these conditions describe what is necessary for a Drive Vehicle Unit (DVU) to receive a stimulus to attempt to change lanes in either direction. The actual lane-changing maneuver will not occur unless a suitable gap exists. The gap acceptance function can be defined by the user, or the default settings can be used.

With regards to car-following, each DVU in the simulation has a target headway. The mean value for target headway is one second by default; however this can be adjusted by the user to match field conditions as necessary. The target headway for each DVU varies around the mean target headway parameter, depending upon the value of certain parameters assigned to the DVU.

In terms of driver behavior, a high aggression value will cause a DVU to accept a smaller headway. Similarly, a high awareness value will affect the use of a longer headway when approaching a lane drop in order to allow DVUs in other lanes to merge more easily.

If not constrained by an approaching junction, a DVU will vary its speed in order to attain its target headway with the DVU in front.

The reaction time of the driver is simulated by basing the calculation of the necessary acceleration/deceleration on the speed at which the DVU in front was traveling at some point in the past.

A default mean reaction time of one second is used, and this is modeled by giving each DVU a memory, so that it carries out with it not only its current speed and position, but a record of its speed and position for a specified number of timesteps in the past. This is referred to as “speed memory” within Paramics. Reducing the driver reaction time is an important factor when considering the throughput of vehicles along a link.

A DVU changes its speed according to its perception of the speed of the DVU in front. These changes are normally smooth, following linear functions, but may be abrupt following the detection of one of two binary signals. These signals are visible brake lights and perceptible acceleration of the DVU immediately ahead. There are therefore three modes of following within the Paramics model, referred to as braking, cruising, and acceleration modes.

For all modes of following, the concept of target point is used. This point is based on a position at an initial distance behind the leading DVU; the target point is then adjusted to improve the car-following behavior.

In addition to the use of an adjusted target point, a bunching acceleration is also used to bring DVUs together rapidly.

In cruising mode, there are five discrete areas, A, B, C, D and E in the headway/velocity-difference phase space. Each of these regions has a separate expression for acceleration. Of these five, three correspond to conditions where the DVU ahead is cruising:

- In Region A, the following DVU has overshot the target point (the headway is less than the target value) and an attempt is made to achieve the target speed as quickly as possible, i.e. as fast as the physical constraints of the DVU allow.

- In Region B, the leading DVU is pulling away from the following DVU.

- In Region C, the DVUs are at a constant separation or coming together.

When the DVU ahead is perceived to be braking (its deceleration is greater than a certain threshold), its perceived speed is decreased by an amount dependent on its maximum deceleration rate. This action models a driver's expectation that if the DVU ahead is braking, its speed in the next time step will be considerably less than at the current time step. The method of application of speed difference and current separation to acceleration ensures that a DVU will over-compensate if the DVU ahead is braking, and that this over-compensation will increase as the distance between the DVUs decreases. This application combined with the time-lag introduced by modeling reaction time results in the shock-wave characteristics as seen typically in highway traffic flow.

However, because the speed of the DVU ahead is predicted, and may have a resultant value of zero, a threshold is used to test whether the following DVU is close enough to be in danger of collision. If not, the acceleration is set to a positive value.

If the DVU ahead is perceived to be accelerating at a high rate, and is more than the following DVUs safe stopping distance away, acceleration is set to the maximum value.

DTA Methodology

The driving force of the Paramics simulation model is an OD matrix applied to a zone map combined with a time-varying profile. This means that the demand on the network between each OD pair can vary in time and can also vary relative to other OD pairs. This leads to a congestion pattern that is also time-variant. To model the route choice decisions that drivers would make based on their knowledge of a time-varying congestion pattern, the user can enable cost table recalculation on a regular basis, perhaps every five minutes of simulation time. The cost recalculation option, when selected, uses mean simulated travel times for links, rather than estimated free-flow travel times. This revised travel time cost can then be injected back into the weighted and factored link cost calculation used previously to create a new routing tree.

Only the route tree for familiar drivers is recalculated at each stage: unfamiliar drivers will still follow the sign-posted routes on the links marked as being major. The ratio of unfamiliar to familiar drivers will determine the damping factor in the feedback control loop: a higher ratio will result in a reduced likelihood of instability.

The justification behind this method is that familiar drivers will have developed experience over time of the true costs of each of the possible routes in the network, and cost feedback and dynamic route recalculation aims to model this phenomenon. It is possible within Paramics to run the model with cost feedback enabled for a period of time, and then save the link costs to file. These link costs can be used as background, or base costs that can be loaded into subsequent runs of the simulation. However, it should be pointed out that cost feedback within Paramics leads to equilibrium only if the time profile of the demand applied to the network is completely flat. For a realistic simulation, it is almost always necessary to model the peaks and troughs of demand, and unless these variations happen at exactly the same time for every OD pair, there will never be a state of equilibrium within the network.

VISSIM Methodologies

VISSIM uses a psycho-physical car-following model for longitudinal vehicle movement and a rule-based algorithm for lateral movements.

The basic idea is the assumption that a driver can be in one of four driving modes:

- Free Driving: No influence of preceding vehicles observable. In this mode, the driver seeks to reach and maintain a certain speed, his individually desired speed. In reality, the speed in free driving cannot be kept constant, but oscillates around the desired speed due to imperfect throttle control.

- Approaching: The process of adapting the driver's own speed to the lower speed of a preceding vehicle; while approaching, a driver applies a deceleration so that the speed difference of the two vehicles is zero in the moment he reaches his desired safety distance.

- Following: The driver follows the preceding car without any conscious acceleration or deceleration. He keeps the safety distance more or less constant, but again due to imperfect throttle control and imperfect estimation the speed difference oscillates around zero.

- Braking: The application of medium to high deceleration rates if the distance falls below the desired safety distance; this can happen if the preceding car changes speed abruptly, or if a third car changes lanes in front of the observed driver.

For each driving mode, the acceleration is described as a result of speed difference, distance, and the individual characteristics of driver and vehicle. The driver switches from one mode to another as soon as he reaches a certain point that can be expressed as a combination of speed difference and distance. For example, a small speed difference can only be realized in small distances, whereas large speed differences force approaching drivers to react much earlier. The ability to perceive speed differences and to estimate distances vary among the driver population, as well as the desired speeds and safety distances. Because of the combination of psychological aspects and physiological restrictions of the driver's perception, the model is called a psycho-physical car-following model.

There are basically two kinds of lane changes in VISSIM, a necessary lane change and a free lane change. In case of a necessary lane change, the driving behavior parameters contain the maximum acceptable deceleration for the vehicle and the trailing vehicle on the new lane, depending on the distance to the emergency stop position of the next connector of the route.

In case of a free lane change, VISSIM checks for the desired safety distance of the trailing vehicle on the new lane. This safety distance depends on its speed and the speed of the vehicle that wants to change to that lane.

In both cases, when a driver tries to change lanes, the first step is to find a suitable gap (time headway) in the destination flow.

DTA Methodology

The DTA procedure in VISSIM is based on the idea of iterated simulation. That means a modeled network is simulated not only once, but repetitively and the drivers choose their routes through the network based on the travel cost they have experienced during the preceding simulations. To model the "learning process", several tasks have to be addressed:

<u>Routes from origins to destinations must be found.</u> VISSIM assumes that not everybody uses the best route but that less attractive routes are used as well, although by a minor portion of the drivers. That means not only the best routes must be known for each OD pair, but a set of routes must be known for each OD pair. Ideally, one would have the set of the *k* best routes but there are no efficient methods to compute this set of routes directly-at least not in a way that makes sense for traffic assignment. The solution adopted in VISSIM is to compute the best paths in each repetition of the simulation and thus find more than one route because traffic conditions change during the iteration. During the iterated simulations, VISSIM builds a growing archive of routes from which the drivers choose.

<u>The routes must have some kind of assessment on which the drivers base their choice.</u> In VISSIM for all routes the generalized costs are computed, i.e. a combination of distance, travel time and "other" costs (e.g. tolls). Distance and costs are defined directly in the network model, but travel time is a result of the simulation. Therefore VISSIM measures travel times on all edges in the network during one simulation so that the route choice decision model in the next simulation can use these values.

The choice on one route out of a set of possible routes is a special case of the more general problem of discrete choice modeling. Given a set of routes and their generalized costs, the percentage of the drivers that choose each route is computed. VISIM uses the logit formulation for this model.

The iteration of the simulation runs is continued until a stable situation is reached. Stable here means that the volumes and travel times on the edges of the network do not change significantly from one iteration to the next. A convergence criteria, either default or user defined determines what a "significant" change is between iterations, similar to what is utilized in static equilibrium assignment in regional models.

AIMSUN Methodologies

The AIMSUN car-following model is based on the P. G. A. Gipps model (Gipps, 1981) which developed as an empirical model consisting of two components, acceleration and deceleration, defined as functions of variables that can be measured. The first represents the intention of a vehicle to achieve a certain desired speed, while the second reproduces the limitations imposed by the preceding vehicle when trying to drive the desired speed.

The AIMSUN car-following model evolved from the Gipps model by making the desired speed a local parameter where the desired speed of vehicle *n* is for the current section of the roadway. Additionally, AIMSUN considers the influence of adjacent lanes so that speeds on adjacent lanes are within reasonable ranges.

The influence of the section grade in the vehicle movement is modeled by means of an increase or reduction of the acceleration and braking capability.

Lane change is modeled as a decision process analyzing the necessity of the lane change, the desirability of the lane change, and the feasibility conditions for the lane change that are also local, depending on the location of the vehicle on the road network.

In order to achieve a more accurate representation of the driver's behavior in the lane changing decision process, three different zones inside a section are considered, each one corresponding to a different lane changing motivation:

<u>Zone 1 is the farthest from the next turning point.</u> The lane changing decisions are governed by the traffic conditions of the lanes involved; the feasibility of the next desired turning movement is not yet taken into account. To measure the improvement that the driver will get on changing lanes several parameters are considered: the desired speed of the driver, speed and distance of the current preceding vehicle, and speed and distance of the future preceding vehicle.

<u>Zone 2 is the intermediate zone.</u> Mainly it is the desired turning lane that affects the lane changing decision. Vehicles who are not driving on a valid lane (i.e. a lane where the desired turning movement can be done) tend to get closer to the correct side of the road where the turn is allowed. In this zone vehicles look for a gap and may try to accept it without affecting the behavior of vehicles in the adjacent lanes.

Zone 3 is the nearest to the next turning point. Vehicles are forced to reach their desired turning lanes, reducing the speed if necessary and even coming to a complete stop in order to make the lane change possible. Also, vehicles in the adjacent lane can modify their behavior in order to allow a gap big enough for the lane-changing vehicle.

Lane changing zones are defined by two parameters: distance to Zone 1 and distance to Zone 2. These parameters are defined in time (seconds) and they are converted into distance whenever it is required for each vehicle at each section using the vehicle desired speed at a section. This means that these distances are then local parameters; their value depending on the current traffic conditions on the section.

The gap-acceptance model used to model give way behavior determines whether a lower priority vehicle approaching a junction can or cannot cross depending on the circumstances of higher priority vehicles (position and speed). This model takes into account the distance of vehicles from the hypothetical collision point, their speeds and their acceleration rates. It then determines the time needed by the vehicles to clear the junction and produces a decision to cross or not which is also a function of the level of risk for each driver. Several vehicle parameters may influence the behavior of the gap-acceptance model, acceleration rate, desired speed, speed acceptance, and maximum give-way time.

DTA Methodology

AIMSUN also has DTA capabilities: both en-route and user equilibrium. The user equilibrium is the same concept as in static planning applications, whereas the en-route assignment uses a combination of link costs, historical paths, and a logit model that assigns a probability to each alternative route between each OD pair depending on the difference of the perceived utilities which are a function of both the link costs and historical path selection.

TransModeler Methodologies

The car-following model in TransModeler is quite complex and dependent on the acceleration rate of the subject vehicle, speed of the subject vehicle, speed of leading vehicle, distance between the subject and leading vehicles, model parameters, and vehicle-specific error term for the car-following regime. Like the other simulation models, headway is an important calibration variable in TransModeler. In TransModeler, the headway is used to determine the boundary between the car-following regime, the emergency regime where the vehicle will apply an appropriate deceleration rate to avoid collision, and the free-flow regime, where the subject's speed is not constrained, or in any way influenced, by the speed or relative position of the vehicle in front.

TransModeler models lane changing behavior in three steps: selection of eligible lanes, lane changing decision-making process, and target lane selection. These steps determine the feasibility, desirability, and safety of a lane change. The selection of eligible lanes will result in a feasible or rational choice set of alternative lanes including as many as three choices: the current lane, and the lanes on the right and left, if they exist. A lane may be excluded from the choice set if the lane use rules in that lane are not compatible with the vehicle's type or if the lane properties restrict lane changes in that direction.

If there is more than one alternative, the selection of the target lane depends on the lane changing regime. The three lane changing regimes are Discretionary Lane Change (DLC), Mandatory Lane Change (MLC), and Forced Lane Change (FLC).

All lane changes are classified as either mandatory or discretionary and a different model and set of parameters is associated with each. Mandatory lane changes are those that are required, for example, to reach an exit ramp or to enter a left turn lane to remain on one's path. A vehicle might make a mandatory lane change to move around an incident or comply with a lane use message. A discretionary lane change is one made in order to achieve a perceived improvement in driving conditions, such as a gain in speed.

A forced lane change is a special case of a mandatory lane change where either an extended period of time has passed where an acceptable gap has not been found or the location before which a lane change must be executed is very near, or both.

Once the MLC, DLC, or FLC model has been applied and both the lane change and the target lane have been decided, the gap acceptance model is applied each time step in the model until an acceptable gap is found and the lane change is completed.

With regards to gap acceptance, when crossing an opposing or conflicting stream, for example making a permitted left turn, vehicles compare their anticipated time to pass through the conflict point with their perception of the time it will take the conflicting vehicles to arrive at that point. If the difference between these times is greater than a minimum acceptable crossing headway, the vehicle will proceed into the intersection. Minimum crossing headway thresholds are likely to be different than those for merging. In the model parameters, the minimum acceptable headways for crossing and merging are defined by a distribution, with headway thresholds varying by segment of the driving population.

DTA Methodology

TransModeler also has DTA capabilities using OD tables from the regional model, either Cube or TransCAD, or other sources. Travel times by time period and network segment can be input from external data or developed by running traffic assignments and traffic simulations. Vehicle paths can also be input from external files including those generated by TransCAD and/or created or edited by analysts. When unexpected delays occur due to incidents, etc. some drivers will change their routes during their trip, which reflects an en-route DTA.

Dynameq DTA Methodology

Dynameq uses a dynamic user equilibrium assignment algorithm, where the equilibrium conditions vary over time based on the temporal profile of demand and congestion in the network. The equilibrium approach to DTA is to allocate vehicles over the best paths on the network for each OD pair so that vehicles leaving the origin at roughly the same time have approximately the same travel times. Dynameq accomplishes this with an iterative method, where each iteration consists of one execution of a traffic simulation and one execution of a path-choice model. The traffic simulator receives time-dependent flow rates from the path-choice model, and simulates the resulting traffic patterns on the network. The simulator then provides time-dependent travel time information back to the path-choice model, which consequently modifies the path choices for the next iteration. The process continues cyclically until converging to an equilibrium, as defined within a threshold defined by the user.

Dynasmart-P DTA Methodology

There currently is limited published information on the methodologies used in Dynasmart. The limited data on the software revealed that Dynasmart uses a dynamic equilibrium assignment, either user optimal or system optimal. Dynasmart also utilizes the First-In, First-Out (FIFO) constraint that other DTA programs utilize as well.

Cube Avenue DTA Methodology

Cube Avenue uses mesoscopic techniques including FIFO constraints where each downstream link maintains a FIFO queue of packets that want to enter but are blocked. Whenever an event on an upstream link says that a packet should move to the next link, the downstream link is queried to determine if it can accept any packets, if not, the packet is removed from the event queue and put at the back of the downstream links blocked queue. Whenever, a packet successfully moves out of a downstream link, the link checks whether it can accept the front packet(s) from the blocked queue. This allows for the representation of queuing in the network. The user can specify whether to examine traffic as individual vehicles or as platoons of multiple vehicles. The user can also specify time increments in terms of minutes or hours and intersection characteristics.

Using these inputs, Cube Avenue computes the lowest-cost path for each vehicle unit, based on its departure time, and computes interactions among vehicle units as they travel through the network. Cube Avenue estimates travel speeds based on vehicle density on road segments during each time increment.

As Cube Avenue is a part of the Citilabs suite, most urban areas utilizing Cube can use the regional transportation model to implement Cube Avenue. The inputs include the roadway network in Cube Voyager format, peak period trip tables, vehicle storage area (generally specified as [distance*lanes]/[average vehicle length]), roadway distance, capacity, and lanes, and traffic signal locations and characteristics.

Cube Avenue uses dynamic equilibrium assignment and loads and tracks the movement of vehicle packets throughout the highway network. The packets can be any size, from individual vehicles up to platoons of 20 or more vehicles.

The outputs can be specified for the time period specified by the user. The outputs include:

- Total traffic volume on a road link

- Total traffic in queue
- Link operating speed and travel time
- Link occupancy/utilization
- Intersection LOS and operating conditions

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