

# Recommendations on Feedback Convergence Methods

*presented to*  
**MWCOG/NC RTPB Travel Forecasting Subcommittee**

*presented by*  
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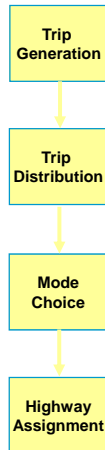
Transportation leadership you can trust.



## Coauthors

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**Gallop Corporation**
- **Liang Long, PhD,**  
**Cambridge Systematics**
- **David Kurth,**  
**Cambridge Systematics**
- **Dalia Leven,**  
**Cambridge Systematics**

## MWCOG/TPB Model Process Version 2.2

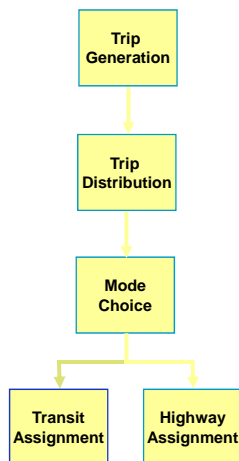


- Four-Step Sequential Model
- Trip Based
- Daily Travel
- Mode Choice – LOV, HOV, TRN
- Highway Assignment

2

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## MWCOG/TPB Model Process Version 2.3

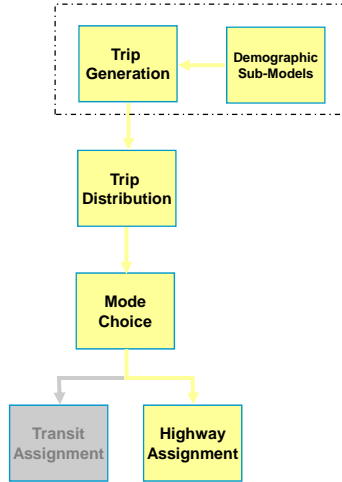


- Four-Step Sequential Model
- Trip Based
- Daily Travel
- Mode Choice – SOV, HOV, TRN by Mode & Access
- Highway Assignment
- Transit Assignment

3

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## MWCOG/TPB Model Process Version 2.2



Version 2.3

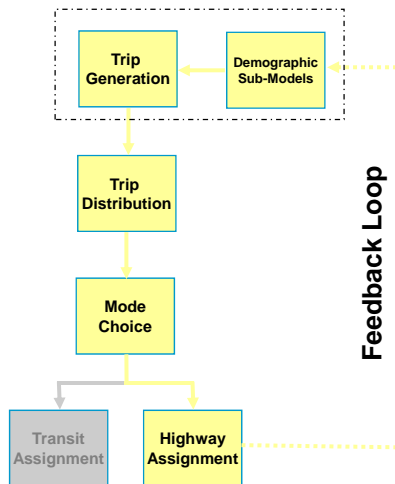
### Demographic Model Output

- Income Level
- Household Size
- Vehicle Availability

4

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## MWVOG Model Process - Feedback



Version 2.3

5

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## Feedback Loop

- **What is the Purpose?**
  - Effects of Congestion
- **How is Implemented?**
  - Loaded Highway Network Skims
- **Details**
  - 3 Iterations Version 2.1c
  - 6 Iterations Version 2.1 D # 50
  - MSA Routine in Assignment

6



## MWCOG/TPB Concerns

- **Model Run Time**
- **Feedback**
  - Pump Prime & 6 Iterations
- **Version 2.2**
  - Base & Conformity Runs
  - 60 Iterations per Assignment x 3 periods
- **Version 2.3**
  - Greater Number of Zones
  - Nested Logit Mode Choice

7



## Objective

*What is the acceptable practice for attaining convergence in the context of the MWCOG/TPB travel demand forecasting model process as well as for other large congested urban area?*

*What are possible metrics that could be considered in determining highway assignment convergence and speed feedback convergence?*

8



## Limited Strategies for Run Time Improvements

- Feedback Loop to Trip Distribution
- Skimming HOV and HOT
- Transit Constraint

9



## Assignment Convergence

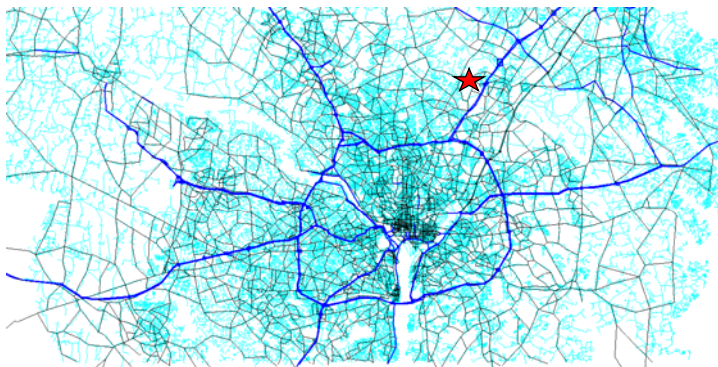
- Applicable to User Equilibrium
- Wardrop's Condition
  - Cost All Paths Equal
- Issue How to Get to Wardrop's Condition
- Measure Relative Gap
  - Path Difference
  - Goal Minimum
- Relationship to Feedback
- Relationship to Model Run Time

10

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## Issues with User Equilibrium

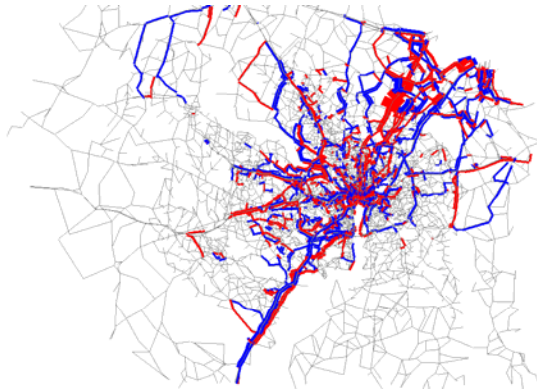
Add One Lane Each Direction  
on US 29 at Howard County Line



11

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## Issues with User Equilibrium



**Add One Lane Each Direction on US 29 at Howard County Line**

**Explain Changes on I-95 to VDOT?**

12

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## Why?

- **Issue is Convergence**
- **Gap=0.01**
- **Convergence Stable Result**
  - **Relative Gap = 0.00001**
    - Florian, Dial, Boyce, Slavin
- **Current State-of-Practice Frank-Wolfe Path Building Algorithm**
- **Very High Number of Iterations to Reach Convergence**

13

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## Reaching Convergence

- **Converge to Stable State Impact on Speed Feedback**
- **New Methods**
  - Link Based
  - Path Based
  - Origin Based
- **Newer Algorithms Acyclic Sub-Network**
  - Algorithm B (Dial)
  - Project Gradient (Florian)
  - Origin User Equilibrium (Slavin)
- **Concern Acyclic Sub-Networks – Route Flows**

14



## Feedback Convergence

- **Over 80% of Large MPO's Feedback Network Time**
- **Convergence = Stability**
- **Criteria?**
  - Minimal Change
  - VMT
  - Link Volumes
  - Trip Table
- **Lack of a Standard**
  - Heuristic Approaches

15





## Feedback Convergence Standard

- **DRCOG**
  - Identify Links with Greater 10% Change
  - When those links are less than 1% of the links in the network, feedback terminates.
- **Encourage MWCOG/TPB Staff**
  - VMT Change Minimal Impacts on Conformity Results
  - Heuristic Approach
  - Not Incorporated into Model Chain

16



## Recommendations

- Apply the method of successive averaging (MSA) procedure for speed feedback but still assign a final trip table to the highway network. This will provide continuity in the final trip tables, the transit assignment applied in Version 2.3, and the final loaded highway network. It will also provide for continuity in application of select link and sub-area extractions.
- Determine a criterion for the number of feedback loops similar in scope to the DRCOG measure. Focusing on the change in VMT and the impacts on conformity could be a possible starting point. It is accepted as a state-of-practice technique for this procedure to use a heuristic approach. The importance will be in documenting the process so it can be understood by decision makers, stakeholders, and other users of the model.

17



## Recommendations (continued)

- Set criteria for relative gap convergence instead of a maximum number of iterations for the highway assignment in the short term.
- Evaluate the impacts on the results of applying a hybrid assignment model which uses an incremental as well as an equilibrium approach. This could involve running the model to reach convergence of close to  $10^{-5}$  as a test, and then using the fixed weights in some form from the results of that assignment for earlier iterations in the model. Then for the final assignment or the later model iterations, applying a user equilibrium assignment with set criteria.
- Follow the developments in acyclic sub-network path based algorithms. In the near term improvements to the algorithms should provide the ability to reduce the number of speed feedback iterations and provide for faster convergence in highway assignment.

18



## 12<sup>th</sup> TRB National Transportation Application and Planning Conference Houston, TX

- Session 14: Traffic Assignment – Divergent Views on Convergence
- Moderator: David Kurth, Cambridge Systematics
- Implications of Consistent Multi-class Link and Route Flows  
Hillel Bar-Gera, Purdue University, Yu Nie, David Boyce, & Yang Liu, Northwestern University
- Application of Accelerated Equilibrium Assignment Methods  
Howard Slavin, Jonathan Brandon, Andres Rabinowicz, & Srinivasin Sundarum, Caliper Corporation
- Changing Assignment Algorithms: the Price of Better Convergence  
Michael Florian & Shuguang He, INRO
- New Empirical Study on Alternative Traffic Equilibrium Algorithms  
Zhong Zhou & Matthew Martimo, Citilabs
- Increasing Precision in Highway Volume Through Adjustment of Stopping Criteria In Traffic Assignment and Number of Feedback  
Kathleen Yu\*, Arash Mirzaei, & Hua Yang, North Central Texas Council of Governments
- Historical Perspectives on Assignment Algorithms  
Robert Dial, Retired

19

<http://teachamerica.com/APP09/index.html>

