Metropolitan Washington Council of Governments
National Capital Region Transportation Planning Board

Review Experience with Equilibrium Assignment

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VHB

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Executive Summary

New techniques are sometimes introduced into the travel demand forecasting process with little long-term follow-up evaluation. Equilibrium assignment, while being more soundly based in theory, has been criticized for being overly sensitive to small network changes resulting in significant changes in volumes on links far removed from the alternatives being studied. If these apparently anomalous behaviors are to be accepted, equilibrium assignment should yield results that are demonstrably better than those produced by other methods.

An objective of this memo was to quantitatively evaluate the performance of different assignment algorithms and critically evaluate the performance of an equilibrium assignment. Recent in-depth analysis of the application of equilibrium assignment has shown instability in the results. When there are small network changes in a localized area, the equilibrium assignment results show large changes in the network for areas that are far from where the network modifications were made. This can be problematic when comparing alternatives and measuring regional benefits.

The incremental capacity restraint (ICR) assignment was used in the primary analysis to compare results with those resulting from an equilibrium assignment. Using a recent model set, it was shown that by increasing the number of equilibrium assignment iterations the discrepancy between the equilibrium results and the incremental assignment results could be decreased. Applying a link refinement post-process as outlined in the National Cooperative Highway Research Project Report Number 255 (NCHRP 255) resulted in a marginal difference in the future forecast between the equilibrium assignment and the eight-iteration ICR assignment.

Even though the equilibrium assignment is theoretically superior to the ICR algorithm, it is troubling that there is instability in the equilibrium assignment and that there are large changes in volumes in areas far from where the network modifications were made. The benefit of an ICR assignment is that results are stable. Increasing the number of iterations can improve the ability of the incremental algorithm to simulate the observed data. Given the large number of iterations required for stability with the equilibrium assignment there is some benefit to applying the incremental algorithm and increasing the number of iterations. The basic issue with respect to equilibrium assignment is related to the stability of the model results. It is much easier to track trips through the network with an incremental approach. This ability to track and understand the travel patterns as well as the added stability are real benefits in planning applications. The drawback is that the ICR assignment did not simulate as well as the equilibrium assignment. Although there was not a lot of difference, a choice has to be made between the need for stability and the accuracy of the model results. With current and future research into finding better ways for equilibrium convergence new techniques may evolve that provide a faster convergence and stability over the current application of the Frank-Wolfe methods.

This memo evaluates equilibrium assignment as well as other techniques. The purpose of this memo is to review issues with the applications of equilibrium assignment, mitigation strategies

for some of these issues, and to provide an update on the state-of-the-practice in highway assignment application at the MPO level.

State of the Practice

TRB recently conducted a survey to determine the state-of-the-practice in travel demand modeling for MPOs. The survey was sent to 381 MPOs across the United States and 228 responded. The MPOs were classified into three categories based on metropolitan area population: small, medium, and large. Metropolitan areas with populations less than 200,000 were categorized as small; those with populations between 200,000 and 1 million were categorized as medium; and those with populations greater than 1 million were categorized as large. MPOs that responded consisted of 116 small MPOs, 74 medium MPOs, and 36 large MPOs. Table 1 provides a summary of the number of surveys and responses.

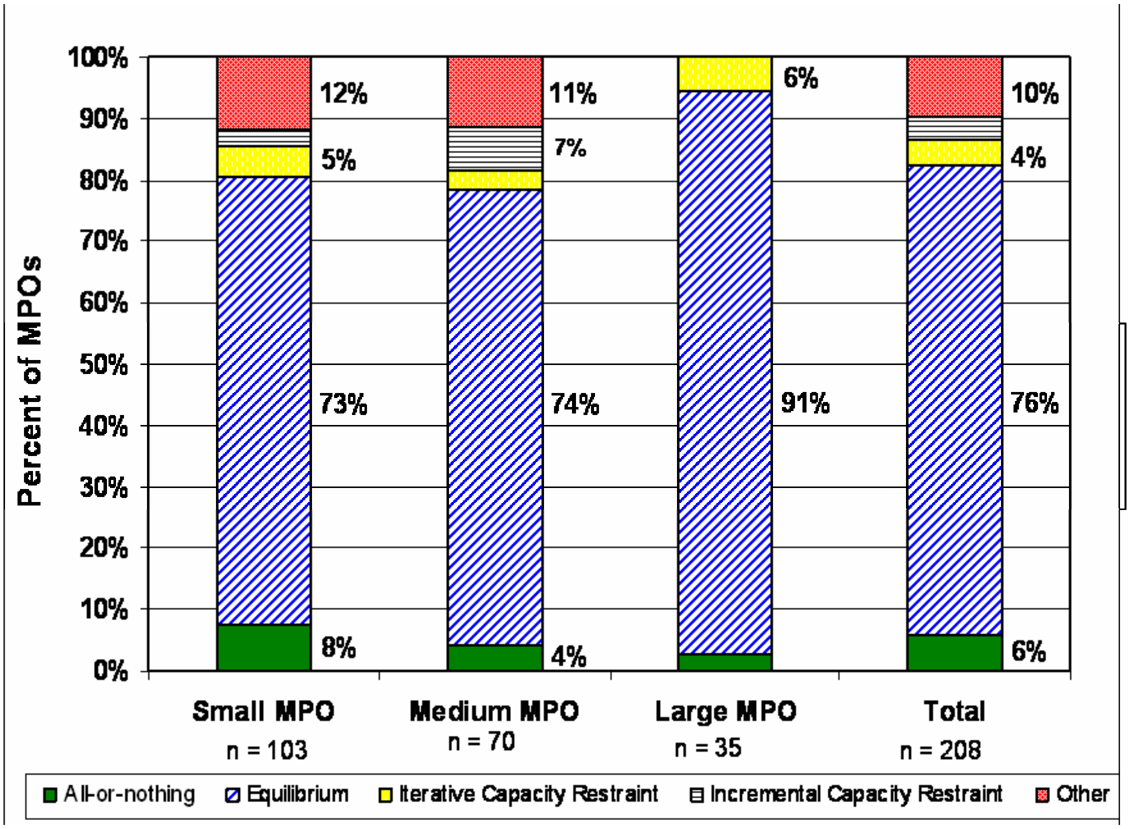
Table 1: Survey Responses

MPO Classification	Surveys Sent	Responses Received	Percent Responding
Small (Population less than 200,000)	205	116	57%
Medium (Population between 200,000 and 1 Million)	133	74	57%
Large (Population greater than 1 Million)	43	36	84%
Total	381	228	60%

For almost all agencies, highway traffic is assigned using an equilibrium method. Figure 1 shows the assignment method used by MPO size. Few agencies were able to report the number of iterations required to achieve closure in equilibrium assignment or the closure tolerance used. Many reported that they used the default values of their software packages. Few agencies had examined equilibrium assignments to see if the results were stable and none of those sampled reported problems such as those noted in studies conducted for FTA.[1] These problems have included issues with stability and consistency with the assignment results.

[1] AECOM, 2005.

Figure 1: Highway Traffic Assignment Method



A recently observed issue with equilibrium assignment is the problem with reaching convergence. For a traffic assignment, equilibrium is reached in a network when for a specific trip interchange the travel time cannot be improved by unilaterally changing paths. The basic assumptions are that each trip chooses the minimum impedance (e.g., travel-time, distance, cost, etc.) and all trips have full information on all impedances for all possible paths. Travelers consistently make the correct decisions regarding route choice, and all travelers have identical behavior. It has been shown that in a highly congested network hundreds of iterations are required until a true state of equilibrium is reached. This problem has been documented in work done by AECOM,[2] Bill Allen,[3] and work that VHB has done. Our work includes reviewing FTA's Summit reports as well as a paper published at the TRB Planning and Applications Conference in 2005 documenting the issue of equilibrium convergence.[4]

There have been several papers published or in the process of being reviewed for publication that deal with the convergence issue with respect to equilibrium assignment. Researchers such as David Boyce and Hillel Bar-Gera have published work on new algorithms that are reported to converge to equilibrium faster and provide a stable solution.[5] Researchers at Caliper Corporation have also funded and refined equilibrium methods that are supposed to reach equilibrium faster

[2] Woodford, 2006.
[3] Allen and Schmitt, 2005.
[4] Goldfarb and Spielberg, 2005.
[5] Boyce, Ralevic-Dekic; and Bar-Gera, 2004.

without hundreds of iterations.[6] Robert Dial recently completed additional research on this topic and was partially funded by Caliper.[7] All of this new research claims and provides examples of applications where by employing a refined equilibrium algorithm the solution was achieved quickly and was stable. Dial and Caliper apply acyclic sub-networks. Boyce and Bar-Gera apply an origin-based assignment algorithm. Caliper researchers have found some issues with the Bar-Gera solution.[8] All of these researchers documented that the Frank-Wolfe method in congested networks will not reach a true equilibrium state and that a new method must be used in congested networks.

Assignment Algorithms

Prior to the application of equilibrium assignment algorithms, many MPOs applied incremental capacity restraint (ICR) assignment algorithms. These algorithms assign part of the trip table, usually proportioned into four equal quarters, and after each increment recalculate travel times and then assign the next proportion until the complete trip table is assigned. This technique was replaced by equilibrium assignment. A key question is what benefit is there to equilibrium assignment given that it replaced the ICR algorithm.

In order to evaluate the performance of different highway assignment algorithms including equilibrium, a travel demand forecast model (TDFM) was used for the Washington, D.C. Metropolitan Area from the year 1990. It was applied to develop forecasts for the year 2000. The most recent land-use data for the year 2000 was obtained and used as the land-use inputs. The objective of this exercise was not to test the validity of the land-use forecast, but to focus on the different assignment techniques. Forecast highway volumes were developed for an approximately 40 mile long portion of the Capital Beltway Corridor (I-495/I-95) in the State of Maryland. Figure 2 shows the geographic location of the study corridor and the major surrounding highway facilities.[9]

The average daily traffic (ADT) and the PM peak hour were simulated and compared to count data for the year 2000. This exercise represented a ten year forecast using the model set from 1990. The observed-to-simulated traffic volumes were compared using a series of statistical measures.

The TDFM set used for this project applied an ICR algorithm as the base assignment algorithm. This was a typical algorithm that was used prior to the wide spread application of the equilibrium algorithm. The ICR algorithm served as the baseline for comparison to the other algorithms.

[6] Slavin, Brandon, and Rabinowicz, 2006.
[7] Dial, 2006.
[8] Slavin, et al, 2006
[9] Maryland State Highway Administration, 2003.

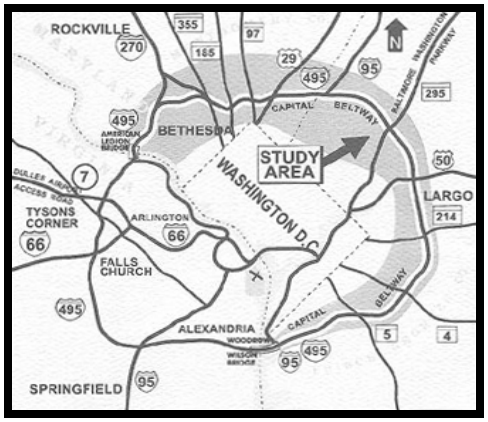


Figure 2

After application of the ICR algorithm, an all-or-nothing (AON) assignment was performed. The purpose in applying the AON assignment for this project was to quantitatively evaluate the assignment as compared to other algorithms and to test the model structure for expected results. By definition, the AON assignment would be expected to over-simulate the freeway links on the Capital Beltway. These links have the highest free-flow speed in the transportation network; therefore, without updating link travel times due to congestion, trips in the vicinity of the freeway would take advantage of the facility's very high performance level. It was expected that the AON assignment results would show higher than observed volumes for the year 2000 and produce the highest traffic volumes when compared to the other assignment algorithms.

The equilibrium assignment algorithm has become the standard algorithm applied for highway assignment by TPB.[10] The primary purpose of this exercise is to quantitatively evaluate the differences in the equilibrium assignment versus the incremental capacity restraint assignment for an actual regional TDFM set. Unlike other tests of the equilibrium assignment, it was not related to an academic exercise or a calibration and validation effort.

The objective of an equilibrium assignment is to disperse traffic across a network so that all paths between origin and destination pairs have equal impedances, usually travel time. The equilibrium assignment process applied here defined equilibrium as when the impedance difference in travel times was less than 3.5 percent across all origin and destination zone pairs. The 3.5 percent was determined to be the acceptable level based on the software vendor's recommendations.[11]

The measures of effectiveness used for the evaluation of the assignments were the percent root mean square error (RMSE), the standard error of regression, and a trend line analysis. The RMSE and the standard error of regression measured the difference in the forecast to the observed count volume. These were the key measures that best quantified which algorithm produced the closest results to the observed count data.

[10] Milone, 2002, pp. 1-4 – 1-8.
[11] Comsis Corporation, 1996, pp. Assign-32.

Daily Traffic Assignment Results

Table 2 summarizes the results of the assignment algorithms for freeway links along the Capital Beltway in Maryland:

Table-2: Summary of Assignment Algorithm Evaluation

Algorithm	RMSE	Standard Error of Regression (1 x 10^3)	Trend Line y-intercept (0,0) Slope
Incremental Capacity Restraint	13.9%	28.15	1.029
All or Nothing	31.6%	64.06	0.078
Equilibrium	11.8%	24.03	1.003

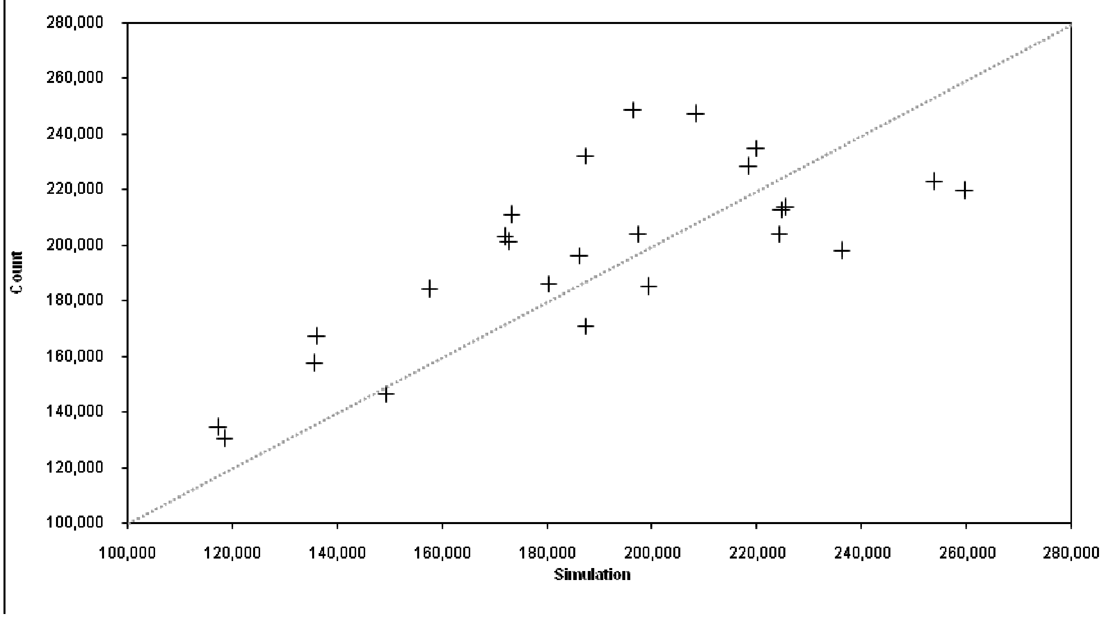
Some general observations are that both the incremental and the equilibrium algorithms matched the count data reasonably well. As expected, the AON assignment did not match the observed data very well. It was expected that the AON assignment would result in an over-simulation. The AON performance was as expected, and this served to confirm that the model assignment process was working. If the AON did not result in an over-simulation, that might have been indicative of a network coding problem. The AON served a dual purpose: the first was as a comparison mark for the other assignment algorithms and the second as a quality control measure.

The remaining two assignment algorithms produced results that were about the same. There was one point where all of the forecast results increased while the count decreased around the interchange at Georgia Ave (MD 97) in Montgomery County. This could be a result of construction on the Beltway in this location during the year 2000. It was not expected that the model network would reflect the effects of construction, even if it were a multi-year project.

The base assignment, which was the ICR algorithm with four iterations, produced reasonable results. The RMSE for this assignment was 13.9 percent. It might have been expected to see some over-simulation on the freeway links since the incremental capacity restraint algorithm is not as responsive to travel time changes as an equilibrium algorithm. The results for this assignment did not show any particular patterns concerning over-simulation on the Capital Beltway. The assignment over-simulated at the Potomac River crossings, but for most of the corridor it under-simulated. The over-simulation in the vicinity of the river crossing could be a result of network supply. There are limited river crossings, including two on the Beltway, and they represent choke points in the system. Based on the Capital Beltway results, it appears that the model was not as successful in simulating the traffic flow from Maryland to Virginia as compared to other links along the Capital Beltway. This is most likely not an assignment issue but more related to trip distribution.

Figure 3 shows the scatter plot for the ICR assignment. The under-simulation is apparent on the plot. Most of the data points fall above the diagonal axis line $y = x$. The trend line analysis resulted in a slope of 1.029. This is reasonably close to 1.0. Overall, the ICR assignment produced relatively good results.

Figure 3: ICR Comparison of Count to Simulation Year 2000



The AON assignment results showed a pattern of over-simulation. The RMSE was 31.6 percent, which was the highest RMSE compared to the other assignment algorithms. Figure 4 shows the scatter plot results from the AON assignment. Except for two data points, all of the links were over-simulated, as could be expected from this type of algorithm. The trend line analysis showed a 0.078 slope showing further evidence of the over-simulation. The Capital Beltway is surrounded by other major and minor arterials, as well as collector facilities, in a dense suburban setting. Given the high speed on the freeway versus the surrounding street network, more trips used the freeway even when this meant backtracking and traveling excessive distances past the destination zone. In reality, when there is a high level of congestion on the Capital Beltway short trips are expected to utilize the arterial network. As expected, the AON algorithm did not represent travel in a mixed urban setting where multiple alternative routes exist. Compared to the ICR assignment, the AON is a poor substitute.

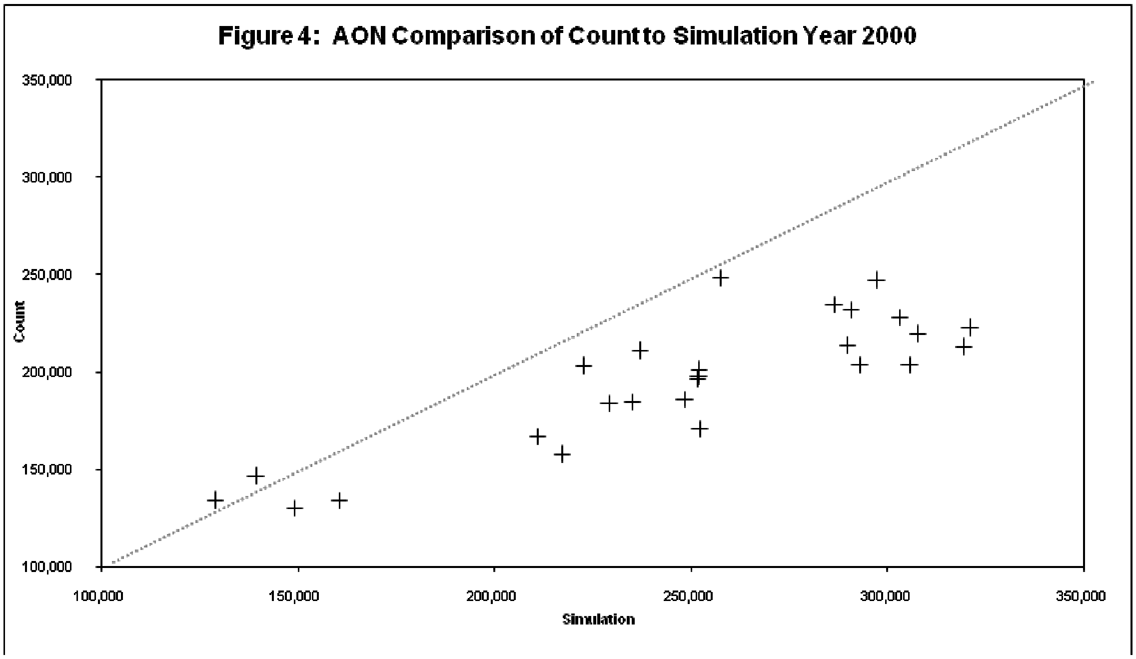
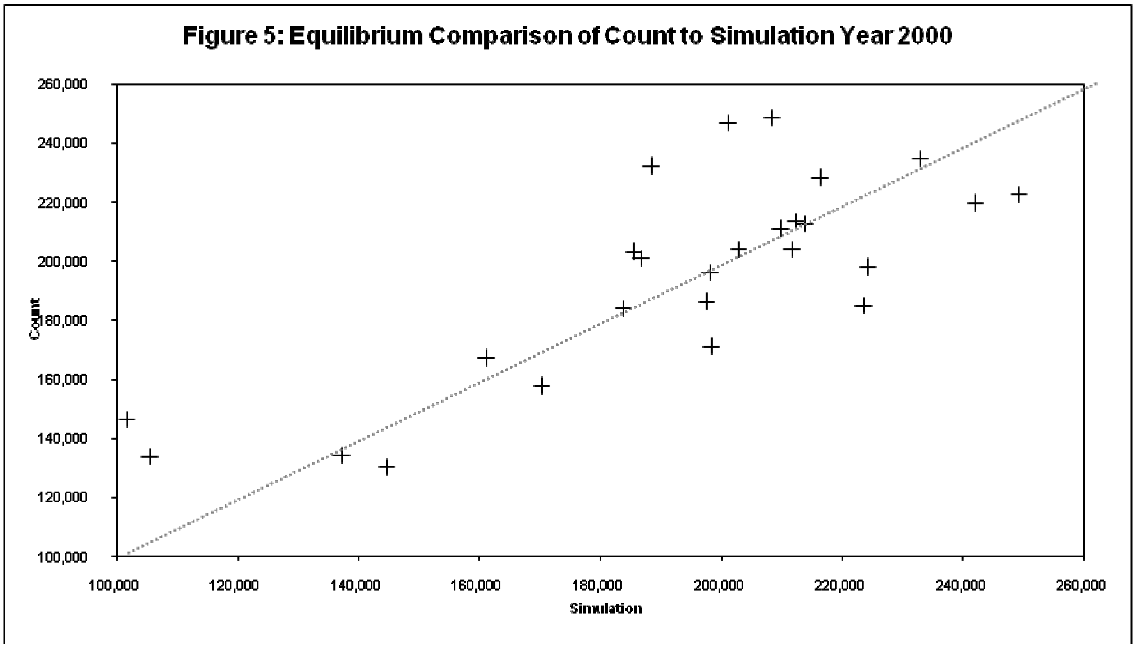


Figure 5 shows the scatter plot results for the equilibrium assignment algorithm. The equilibrium algorithm is designed to spread traffic across all competing routes. It stops once the sum of all impedances (in this case travel times) on the links are equal within a given tolerance level.

The equilibrium assignment algorithm showed the lowest RMSE for all of the tested algorithms. The RMSE was 11.8 percent, and the trend line analysis resulted in a slope of 1.003. Although marginally better than the base incremental assignment algorithm, the equilibrium assignment did take a longer time to execute. The amount of time was about ten minutes longer, or roughly a 30 percent increase. Given the improved results, the time difference was not substantial. As with all of the assignments, the Potomac River crossings were over-simulated by the equilibrium assignment. For the river crossings, it did not perform any worse or better than the incremental algorithm.

Time of Day Assignment Evaluation

As part of this project, a PM peak period assignment was performed using both an incremental capacity restraint assignment and an equilibrium assignment. The time-of-day (TOD) method applied for this project factored the trip tables by purpose prior to the final highway assignment. The factors were taken from the latest version of the regional travel demand model. The PM peak period was defined as from 4:00 pm to 7:00 pm.



The results of the assignments were then used to derive PM peak hour volumes. A simple peak hour to peak period factor was applied to each link volume. The factor was taken from the permanent count stations along the Capital Beltway. The data showed that for the year 2000, the peak hour was 34 percent of the peak period. This was consistent for all of the stations.

Table 3 presents a summary of the TOD results. For this evaluation, the RMSE was the key measure used to evaluate the results. The RMSE for the incremental capacity restraint assignment were 51.2 percent for the Inner Loop of the Capital Beltway and 48.5 percent for the Outer Loop. The RMSE using the equilibrium assignment was somewhat better. The Inner Loop RMSE with the equilibrium assignment was 20.9 percent and for the Outer Loop was 24.5 percent. The equilibrium algorithm performed much better when doing a peak period assignment as compared to the daily assignment. This could be due to a variety of factors. One major factor could be the level of network saturation in the peak period versus the daily.

Table 3: RMSE Summary of TOD Assignment (by direction)

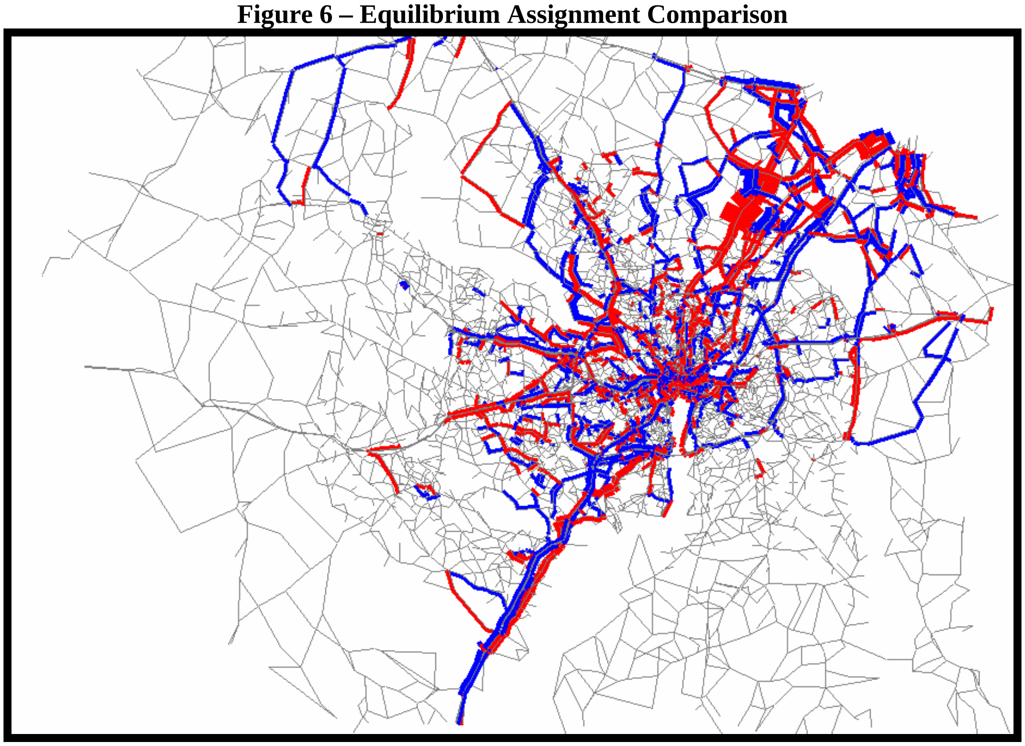
Algorithm	**Inner Loop**	**Outer Loop**
Incremental Capacity Restraint	51.2%	48.5%
Equilibrium	20.9%	24.5%

Equilibrium Assignment Issues

The purpose of this exercise was to quantitatively evaluate the performance of different assignment techniques commonly used in travel demand forecast models. The equilibrium algorithm did produce the best results, but there are several outstanding issues related to the closure of the equilibrium assignment. Even when the time between all travel paths is less than 0.1 percent, small changes in the network result in large discrepancies across the network. Localized network changes, even as small as adding lanes to a specific facility, can result in different assignment results far from the network change.

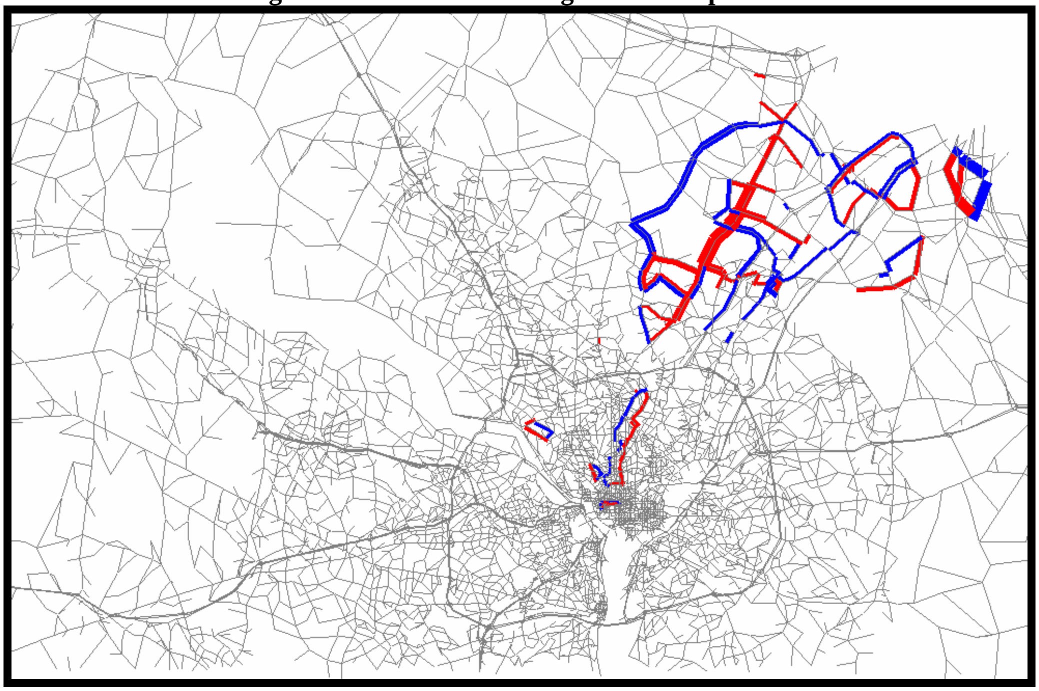
Figures 6 and 7 show an example of this issue. Using a more recent model set, a lane was added in each direction to six miles of US 29 in Howard and Montgomery Counties. Figure 6 shows the difference between the network assignments using an equilibrium assignment. The gap was 0.1 percent and the assignment took over 30 iterations to be completed. Figure 7 used the same two networks, but applied an ICR algorithm. Each increment assigned 12.5 percent of the daily trip table. Figure 6 shows changes in link volumes throughout the greater metropolitan area, even in areas far from the network change. Figure 7 shows the change in assignment for predominately the area surrounding where the network change was made. Figure 7 shows a change pattern that is much more logical and easier to comprehend. The network differences far from the impacted area in Figure 6 are challenging to accept and do not make logical sense. The ICR algorithm does provide some stability in the assignment between the two test alternatives.

Figure 6 – Equilibrium Assignment Comparison



The question is whether the benefits of applying an equilibrium assignment outweigh the instability of the results. It is problematic that small changes in the network produce differences in the results for areas far from where the network changes were made. Increasing the number of iterations can be useful for both algorithms in order to improve their performance. For equilibrium assignment to show stability the gap must be less than 0.1 percent. With this small of a gap, the number of iterations for a large network will be well over 30. For an ICR algorithm, a greater number of iterations can result in closer replication of the equilibrium assignment with less network instability.

Figure 7 – Incremental Assignment Comparison



A limited comparison of simulated and observed data was made between an ICR algorithm with eight iterations and an equilibrium assignment with a gap of 0.1 percent. This comparison was done using the more recent model set for the Washington, D.C. metropolitan area while looking at two different cutlines composed of a total of 12 facilities. The first cutline served more local traffic while the second cutline served predominately commuter traffic. The resulting difference in the RMSE between an equilibrium assignment with a gap equal to 0.1 percent and an incremental capacity restraint algorithm with eight iterations was found to be very small. The RMSE for both cutlines combined was 21.4 percent for the incremental capacity restraint assignment and 20.6 percent for the equilibrium assignment. After applying a post-process link refinement routine the difference in individual facilities at the ADT level was less than or equal to one percent. Table 4 summarizes the results.

As part of on-going transit project planning work, AECOM has come to the same conclusion. Using the TPB model set as well as the Hampton Roads Planning District Commission (HRPDC) model set for the Hampton Roads area, AECOM has shown that for equilibrium to be reached in congested networks greater than 500 iterations are required, and only with this many iterations can stability be achieved when applying an equilibrium assignment. These issues with equilibrium assignment have only been brought to light as a result of transit analysis for New Starts programs. Using FTA's Summit software abnormalities in the highway assignment have been discovered while testing different transit alternatives and calculating user benefits. Prior to these exercises issues with convergence and the instability of the assignment went unnoticed.

Table 4: Summary of Cutline Results for Count Year

Facility	**2000 Count (ADT)**	**Incremental Capacity Restraint 8 iterations (ADT)**	**Equilibrium gap=0.1% (ADT)**
US 29	45,000	63,112	59,433
I-95	165,000	166,528	169,102
MD 216	16,100	11,667	12,821
US 1	34,000	38,609	35,065
MD 108	15,000	27,107	22,708
Washington Grove Ln	19,300	17,175	18,119
Shady Grove Road	49,500	41,581	41,581
Redland Road	22,000	31,504	34,248
Southlawn Lane	2,000	1,553	5,926
MD 28	52,000	51,452	42,881
Baltimore Avenue	5,300	11,863	13,110
MD 586	35,200	30,203	30,753

One technique used to address this issue with stability and equilibrium assignment is a type of hybrid approach. The Baltimore Metropolitan Council (BMC) uses this hybrid equilibrium assignment method. The technique applies an equilibrium assignment for a designated base year, and then uses the resulting weights from each of the assignment iterations for future year assignments. This is essentially an iterative assignment and the averaging proportions are derived from an equilibrium assignment and the linear optimization of the λ (lambda) function. This process was tested using the model set detailed earlier in this memorandum. It was found to be more stable than the equilibrium assignment but not as stable as the incremental assignment. The results of this evaluation are summarized in Table 5.

The resulting RMSE for the hybrid approach was 23.4 percent compared to 20.6 percent for the equilibrium assignment and 24 percent for the eight iteration incremental assignment. This technique does address the concerns for having all paths in the network approaching a state of equilibrium, but for the additional level of work, it does not seem to add much benefit.

Table 5: Summary of Cutline Results for Count Year with Hybrid Equilibrium Assignment				
Facility	**2000 Count (ADT)**	**Incremental Capacity Restraint 8 iterations (ADT)**	**Equilibrium gap=0.1% (ADT)**	**Hybrid Equilibrium with Fixed Weights From Base Equilibrium Assignment (ADT)**
US 29	45,000	63,112	59,433	63,565
I-95	165,000	166,528	169,102	155,988
MD 216	16,100	11,667	12,821	13,179
US 1	34,000	38,609	35,065	37,894
MD 108	15,000	27,107	22,708	23,502
Washington Grove Ln	19,300	17,175	18,119	19,567
Shady Grove Road	49,500	41,581	41,581	42,767
Redland Road	22,000	31,504	34,248	34,043
Southlawn Lane	2,000	1,553	5,926	9,800
MD 28	52,000	51,452	42,881	45,929
Baltimore Avenue	5,300	11,863	13,110	13,537
MD 586	35,200	30,203	30,753	32,520

Conclusion

An objective of this memo was to quantitatively evaluate the performance of different assignment algorithms and critically evaluate the performance of an equilibrium assignment. Recent in-depth analysis of the application of equilibrium assignment has shown instability in the results. When there are small network changes in a localized area, the equilibrium assignment results show large changes in the network for areas that are far from where the network modifications were made. This can be problematic when comparing alternatives and measuring regional benefits.

The incremental capacity restraint assignment used in the primary analysis was composed of four iterations. Each of the four iterations assigned an additional 25 percent of the trip table to the network and then recalculated the shortest path. Using a more recent model set, it was shown that by increasing the number of iterations the discrepancy between the equilibrium results and the incremental assignment results could be decreased. Applying a link refinement post-process as outlined in the National Cooperative Highway Research Project Report Number 255 (NCHRP 255) resulted in a marginal difference in the future forecast between the equilibrium assignment and the eight-iteration ICR assignment.

Even though the equilibrium assignment is theoretically superior to the ICR algorithm, it is troubling that there is instability in the equilibrium assignment and that there are large changes in volumes in areas far from where the network modifications were made. The benefit of an ICR assignment is that results are stable. Increasing the number of iterations can improve the ability of the incremental algorithm to simulate the observed data. Given the large number of iterations required for stability with the equilibrium assignment there is some benefit to applying the incremental algorithm and increasing the number of iterations. The basic issue with respect to equilibrium assignment is related to the stability of the model results. It is much easier to track trips through the network with an incremental approach. This ability to track and understand the travel patterns as well as the added stability are real benefits in planning applications. The drawback is that the ICR assignment did not simulate as well as the equilibrium assignment. Although there was not a lot of difference, a choice has to be made between the need for stability and the accuracy of the model results. With current and future research into finding better ways for equilibrium convergence new techniques may evolve that provide a faster convergence and stability over the current application of the Frank-Wolfe methods.

While new methods of reaching convergence with equilibrium algorithms are being developed for macro-level assignments, the move to more meso-level and micro-level traffic assignments is the future. This is true even within the framework of the region travel demand forecast model. TPB should allocate time and resources to examining these evolving methods and how they can be applied.

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